Introduction to Information Retrieval

http://informationretrieval.org

IIR 6&7: Vector Space Model

Hinrich Schütze

Institute for Natural Language Processing, University of Stuttgart

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Models and Methods

1. Boolean model and its limitations (30)
2. Vector space model (30)
3. Probabilistic models (30)
4. Language model-based retrieval (30)
5. Latent semantic indexing (30)
6. Learning to rank (30)

Take-away

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- tf-idf weighting: Quick review of tf-idf weighting

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- Vector space model – represents queries and documents in a high-dimensional space.

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- tf-idf weighting: Quick review of tf-idf weighting
- Vector space model – represents queries and documents in a high-dimensional space.
- Pivot normalization (or "pivoted document length normalization"): alternative to cosine normalization that removes a bias inherent in standard length normalization

Outline

1. tf-idf weighting
2. Vector space model
3. Pivot length normalization

Binary incidence matrix

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ANTHONY	1	1	0	0	0	1	
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- Relevance does not increase proportionally with term frequency.

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$$\mathrm{w}_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d} & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$$

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- $\mathrm{tf}_{t,d} \rightarrow \mathrm{w}_{t,d}$:
  $0 \rightarrow 0$, $1 \rightarrow 1$, $2 \rightarrow 1.3$, $10 \rightarrow 2$, $1000 \rightarrow 4$, etc.
- Matching score for a document-query pair: sum over terms t in both q and d:
  tf-matching-score$(q, d) = \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$ □

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- ...we also want to use the frequency of the term in the collection for weighting and ranking.

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- $[\log N/\mathrm{df}_t]$ instead of $[N/\mathrm{df}_t]$ to "dampen" the effect of idf

Examples for idf

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$$\mathrm{idf}_t = \log_{10} \frac{1{,}000{,}000}{\mathrm{df}_t}$$

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

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- For example, in the query "arachnocentric line", idf weighting increases the relative weight of ARACHNOCENTRIC and decreases the relative weight of LINE.
- idf has little effect on ranking for one-term queries. □

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  - ... increases with the rarity of the term in the collection. (inverse document frequency component)

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Caesar	8.59	2.54	0.0	1.51	0.25	0.0	
Calpurnia	0.0	1.54	0.0	0.0	0.0	0.0	
Cleopatra	2.85	0.0	0.0	0.0	0.0	0.0	
mercy	1.51	0.0	1.90	0.12	5.25	0.88	
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- Each vector is very sparse - most entries are zero.

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Queries as vectors

- Key idea 1: do the same for queries: represent them as vectors in the high-dimensional space
- Key idea 2: Rank documents according to their proximity to the query
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- Recall: We're doing this because we want to get away from the you're-either-in-or-out, feast-or-famine Boolean model.
- Instead: rank relevant documents higher than nonrelevant documents

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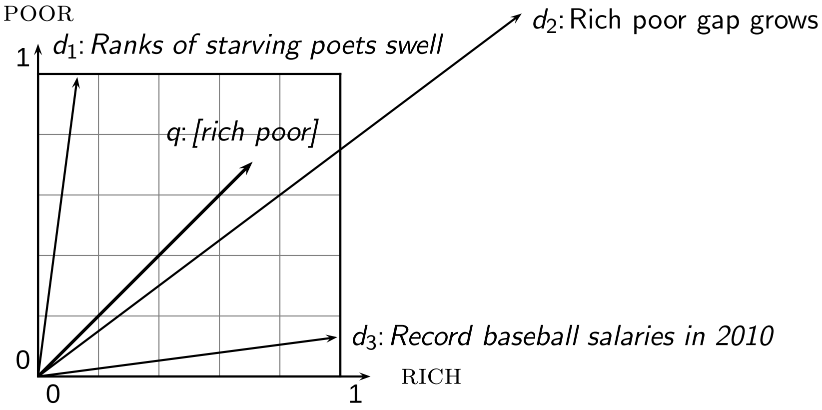
- First cut: (negative) distance between two points
- (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

Why distance is a bad idea



The Euclidean distance of $\vec{q}$ and $\vec{d}_2$ is large although the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.

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- $\rightarrow$ do ranking according to cosine

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- cosine similarity = dot product of length-normalized vectors

Cosine similarity illustrated

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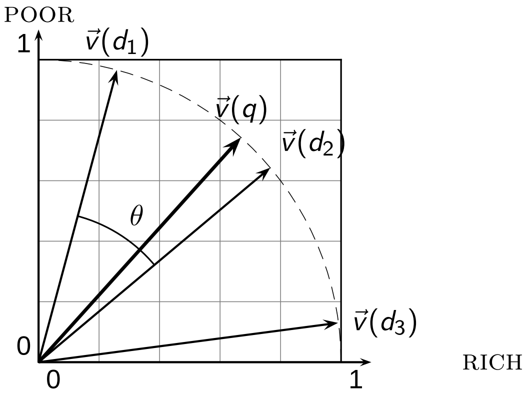

Components of tf-idf weighting

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5 + \frac{0.5\times\mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t\in d}(\mathrm{tf}_{t,d}))}$				

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Best known combination of weighting options

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- Isn't it bad to not idf-weight the document?

tf-idf example

- We often use different weightings for queries and documents.
- Notation: ddd.qqq
- Example: lnc.ltn
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tf-idf example: lnc.ltn

Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto										
best										
car										
insurance										

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto	0									
best	1									
car	1									
insurance	1									

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best	1					0				
car	1					1				
insurance	1					2				

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insurance	1	1				2	1.3			

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best	1	1	50000			0	0			
car	1	1	10000			1	1			
insurance	1	1	1000			2	1.3			

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best	1	1	50000	1.3	1.3	0	0	0	0	
car	1	1	10000	2.0	2.0	1	1	1	0.52	
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	

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$\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$
$1/1.92 \approx 0.52$
$1.3/1.92 \approx 0.68$

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insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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Final similarity score between query and document: $\sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08$

Outline

1. tf-idf weighting
2. Vector space model
3. Pivot length normalization

A problem for cosine normalization

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  - d_3: a short document on anti-doping rules at the 2004 Athens Olympics
- What ranking do we expect in the vector space model?

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- This removes the unfair advantage that short documents have.
- Singhal's study is also interesting from the point of view of methodology.

Predicted and true probability of relevance

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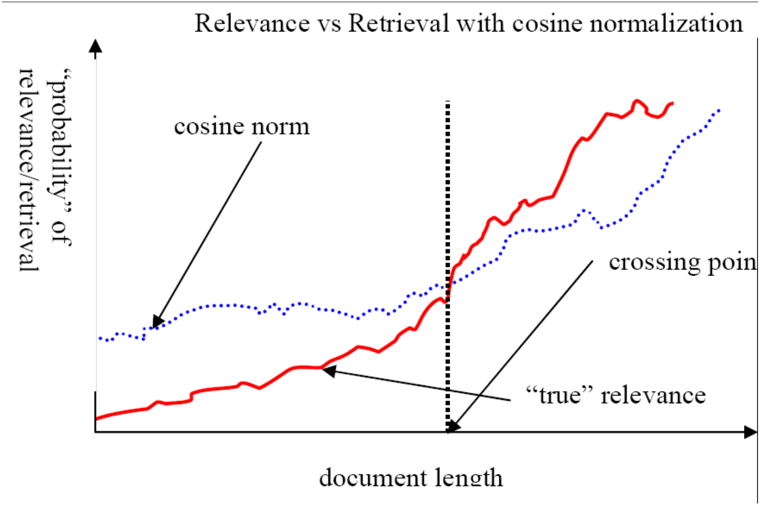

source: Lillian Lee

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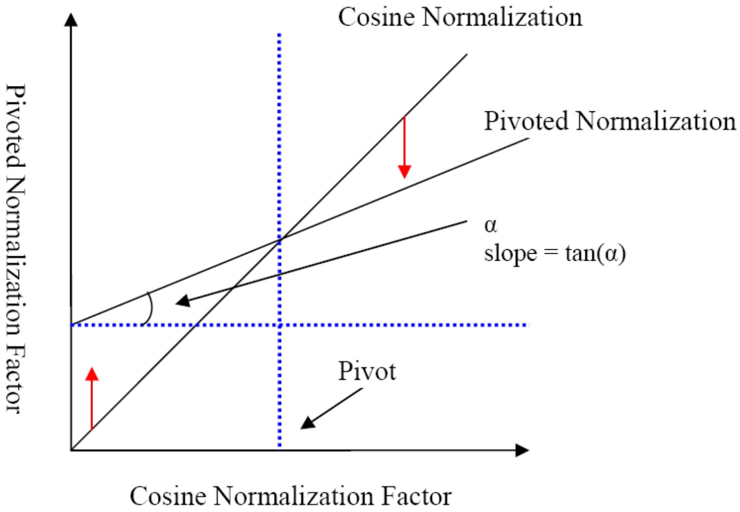

source:
Lillian Lee

Pivoted normalization: Amit Singhal's experiments

Pivoted normalization: Amit Singhal's experiments

Cosine	Pivoted Cosine Normalization				
	Slope				
	0.60	0.65	0.70	**0.75**	0.80
6,526	6,342	6,458	6,574	**6,629**	6,671
0.2840	0.3024	0.3097	0.3144	**0.3171**	0.3162
Improvement	+ 6.5%	+ 9.0%	+10.7%	**+11.7%**	+11.3%

(relevant documents retrieved and (change in) average precision)

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- Represent each document as a weighted tf-idf vector
- Represent the query as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
  - Alternatively, use pivot normalization
- Rank documents with respect to the query
- Return the top K (e.g., $K = 10$) to the user

Take-away

- tf-idf weighting: Quick review of tf-idf weighting
- Vector space model – represents queries and documents in a high-dimensional space.
- Pivot normalization (or "pivoted document length normalization"): alternative to cosine normalization that removes a bias inherent in standard length normalization

Resources

- Chapters 6 and 7 of Introduction to Information Retrieval
- Resources at http://informationretrieval.org/essir2011
  - Gerard Salton (main proponent of vector space model in 70s, 80s, 90s)
  - Exploring the similarity space (Moffat and Zobel, 2005)
  - Pivot normalization (original paper)